Statistics 5444: Homework 6

For each homework assignment, turn in at the beginning of class on the indicated due date. Late assignments will only be accepted with special permission. Write each problem up *very* neatly (LaTeX is preferred). Show all of your work.

Problem 1

In this problem you will simulate the St. Petersburg Paradox. For a reminder of the statement of the problem: consider a game, where you flip a fair coin ($p = 1/2$) until the first occurrence of a head. Let k denote the number of trials until the first head. For some value of k, your winnings will be 2^{k-1} (with probability $1/2^k$). Simulate your earnings 10,000 times. Plot the results of average winnings over the 10,000 trials. The Y-axis will have expected winnings on it, the X-axis will have the number of games for which the average was computed.

Part 2

After looking at your results, specify how much you might be willing to spend to play this game (you only get to play once).

Part 3

The notion of utility (akin to loss) measures how much something is actually worth to you. In the context of this problem, the utility of playing this game is the amount of money it would be worth for the opportunity to win 2^{k-1} (for random k) dollars. Let's motivate the idea of utility a little more. Fathom the idea of winning 100 billion dollars and also fathom the idea of winning 10 Billion dollars. Are these dollar amounts similar? Clearly no. But the utility of the money might be more similar than the exact dollar amounts. That is, under your current lifestyle, it might not matter if you have 10 Billion dollars or 100 billion. Under your current lifestyle, you probably couldn't spend either. A utility function measures the relative worth of something (like money). Some economists will claim that money is best represented by a log utility function. That is , if you have 10 dollars, it's relative utility is log(10) dollars. Under this concept, we can calculate the Expected Utility of the above game. Under the Log utility function, we have

$$E[U] = \sum_{k=1}^{\infty} \log(2^{k-1})/2^k.$$

Evaluate this quantity and then express, under the log utility function, how many *actual* dollars you would spend to play the game.

Problem 2

Prove that the under a proper prior that any Bayes estimator is always admissible.

Problem 3

Consider the loss function $L(\theta, e)$, which measures the loss incurred by estimating θ by $e(X)$, where X is observed data. Let $p(\theta|X) \propto f(X|\theta)\pi(\theta)$ be the posterior distribution (under *proper* prior $\pi(\theta)$). Denote the Bayes Rule by e^π, where

$$e^\pi = \operatorname*{argmin}_e \int L(\theta, e(X))p(\theta|X)d\theta.$$

Show that if e^π has a constant risk (over θ), it is minimax.